Using the Doppler Shift of a He-Ne Laser to Measure Non-relativistic Speeds

Tristan Larkin

May 22, 2025

Abstract

The Doppler effect is a well-understood phenomenon that causes an apparent shift in the frequency of light when there is a relative velocity between the observer and the source of the light. Astronomers and cosmologists have used the Doppler shift to measure the speed of galaxies moving away from the Milky Way based on the red-shift of the light coming from galaxies moving away from the Earth. By reflecting a laser off a moving object and letting it interfere with the source beam, a spectrum analyzer can pick up the beat frequency produced by the interference. This report shows that the interference patterns formed from letting Doppler-shifted light interfere can be used to measure the instantaneous velocity of moving tabletop objects. Our results using the Doppler shift of an object moving longitudinally away from the laser had a 21% error, the longitudinal speed of a slow wheel had a 1.6% error, and the transverse speed of a fast wheel had a 14% error when compared to our measurements of each objects average speed over time. The ability to measure the instantaneous speed of an object up to the resolution of a spectrum analyzer is beneficial for analyzing systems where accurately knowing the transverse or rotational speed of an object at all times in an experiment is vital.

Introduction

Good measurements of physical quantities are vital to quality research. Getting a good measurement can be a non-trivial task in some cases. Light is often used to gather information about things that are difficult to measure, which is the case for many microscopes and telescopes. In modern science, using light outside the visible spectrum is common, such as the James Webb telescope, which has a wavelength on the order of meters [1]. For tabletop measurements, it is convenient to have visible light for setting up the experiment. A He-Ne laser is a strong candidate for studying tabletop objects at relatively low speeds since they produce light at [2].

The Doppler effect was discovered in 1846 [3] and has since become an important phenomenon for studying the speed of waves. The most general form of the Doppler effect when working with a light source and observer moving at non-relativistic speeds is

$$\nu = \nu_0 \frac{c - v_{obs}}{c - v_{src}} \tag{1}$$

where ν_0 is the original frequency of the light in the light source's reference frame, c is the speed of light (in this version of the formula, it requires a sign to show what direction the light is propagating), v_{src} is the velocity of the light source, v_{obs} is the velocity of the observer, and ν is the Doppler shifted frequency. Since this is the 1D, non-relativistic Doppler effect, all velocities in the equation have to be in the same direction or must be exchanged with the components of the velocity in the direction.

The Doppler effect causes light sources moving away from an observer to be red-shifted; if it moves towards the observer, the light gets blue-shifted, as shown in Figure 1. This effect revealed that the observable universe is expanding as it was observed that most other galaxies are red-shifted.

Directly measuring the frequency of visible light requires very good equipment, and it is easier to measure lower frequencies. When two beams of light with close frequencies interfere, a beat frequency is produced [4]. The beat frequency that is produced is given by

$$\nu_{beat} = |\nu_1 - \nu_2| \tag{2}$$

where ν_1 and ν_2 are the two frequencies of light that are interfering and ν_{beat} is the resulting beat frequency. The beat frequency resulting from the interference of light from a He-Ne laser with and with-out a Doppler shift from being scattered off a moving object is within the range a photodetector captures and a spectrum analyzer measures. Our experimental procedure involves using these attributes by creating an interferometer that splits the beam and Doppler-shifts one and merges them back together to get the interference pattern we need to find the speed of the reflected surface.

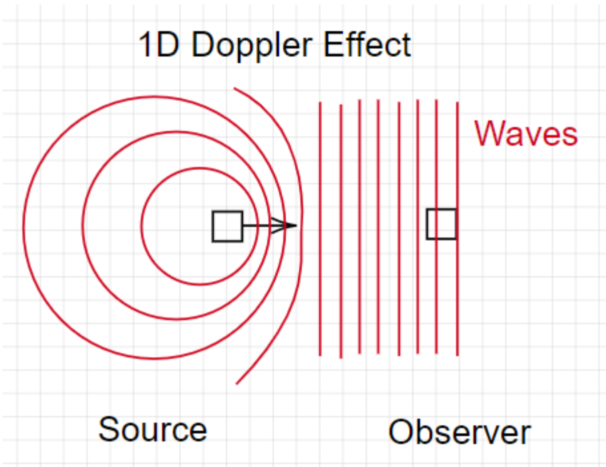

Figure 1: Visual example of the Doppler effect. Shows that wavefronts on one side of a wave source get closer together (blue-shifted) and farther apart on the other side (red-shifted).

Methods

This report examines three systems to test the hypothesis: a stationary slow-spinning wheel, a slow-moving train moving away from the laser, and a stationary fast-moving wheel. In all situations, we use the Doppler effect and beat frequency to find the speed of the object in question, but the exact experimental procedures vary. In all cases, a photodetector will be connected to a spectrum analyzer to look for the beat frequency produced by interfering beams. To find what signals are caused by the interference of the beams, compare the signals seen with only one beam to the signal with both beams. Many cycles of the object's motion will also be measured and times to get the average speed of the object.

Slow Wheel

The experimental setup for measuring the speed of the slow wheel, Figure 2, is an interferometer where one leg reflects off a mirror and the other is scattered off a moving wheel. The systems should be set up so the wheel can be moved so the laser hits the wheel at different points. The horizontal distance from the center of the wheel to where the laser hits the wheel will be called a. We took measurements at five different values of a. Also, we measured the slow wheel directly by timing individual rotations many times to get a value to compare the Doppler effect measurement against.

Slow Train

Determining the speed of an object moving away from the laser will use Eq. 3. Figure 3 shows that the setup is also very similar, using the same interferometer but with the slow wheel switched out with the train. To get the average speed, let the train run on a known length of track for multiple runs and time them. This version of the experiment comes with some extra challenges because the train is moving, so there must be enough track to get up to full speed in all measurements.

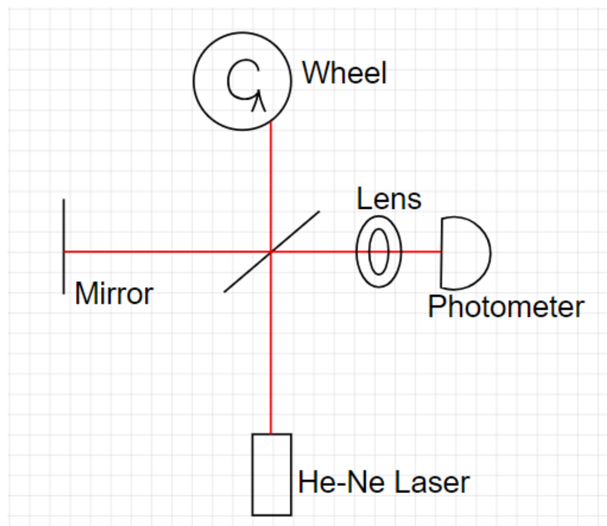

Figure 2: This is the setup for the slow wheel experiment. The wheel and mirror are covered in reflective tape to scatter the light. The wheel is rotating at an unknown angular velocity ω.

Fast Wheel

When an object is moving very fast, the Doppler shift, as seen in the slow wheel and slow train experiments, will be larger, which means the beat frequency will also be larger. Instead of interfering with the Doppler-shifted light with the unshifted laser, this experiment looks at the beat frequency caused by two beams that were Doppler-shifted a different amount, Figure 4.

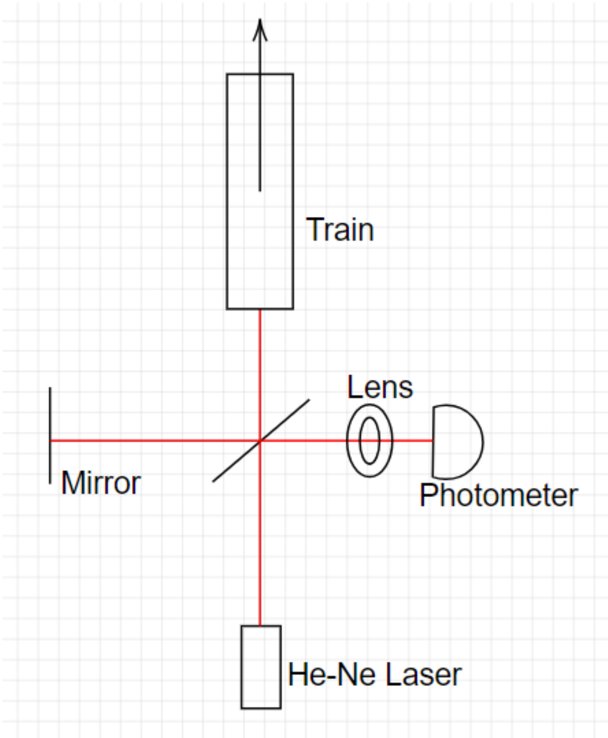

Figure 3: This is the setup for the train experiment. The train and mirror have reflective tape to scatter the light. The train is moving at an unknown velocity v_{train} away from the beam splitter.

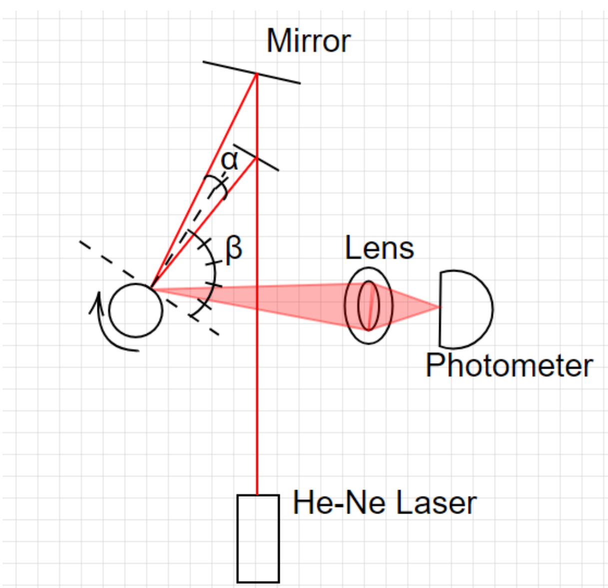

Figure 4: This is the setup for the fast wheel experiment. The wheel has an unknown angular velocity ω. α is the angle between the two beams, and β is the angle between the line tangent to the surface of the wheel and the bisector of α.

To measure the speed of the wheel directly, we marked a section of the reflective wheel with a black dry-erase marker. By sending that data from the photodetector (as shown in Figure 4) to an oscilloscope, we can find the rate at the wheel spins by looking at the time step between when the oscilloscope reads voltage drops. The value measured here will be referred to as the actual speed of the fast wheel.

Data

Data is collected from the spectrum analyzer present in each setup. To ensure we measured the beat frequency resulting from beam interference, we checked the spectrum analyzer with both beams and with a single beam, as shown in Figure 5. The uncertainty of the data collected from the spectrum analyzer is half the full-width half-max.

Slow Wheel

The Doppler shifted data gathered from the spectrum analyzer is listed in Table 1. Figure 6 shows an example of the raw information used to collect the data in Table 1. When measured directly by timing multiple rotations, the angular speed of our wheel is 1.24 ± 0.01 rad/s.

a (cm)	3.0	3.5	4.0	4.5	5.0
ν_{beat} (kHz)	108.	126.	148.	165.	181.
ω (rad/s)	1.24	1.24	1.24	1.24	1.24

Table 1: The data recorded for the slow wheel experiment. a is the distance from the center of the slow wheel. ν_{beat} is the frequency recorded by the spectrum analyzer. ω is the calculated angular speed of the wheel. Uncertainties are omitted here for space.

Slow Train

The direct measurement of the train was 0.19 ± 0.01 m/s. When using the Doppler effect, we took 20 data points and found the mean and standard deviation of the beat frequencies, Figure 7. The beat frequency of the moving train was 460 ± 20 kHz.

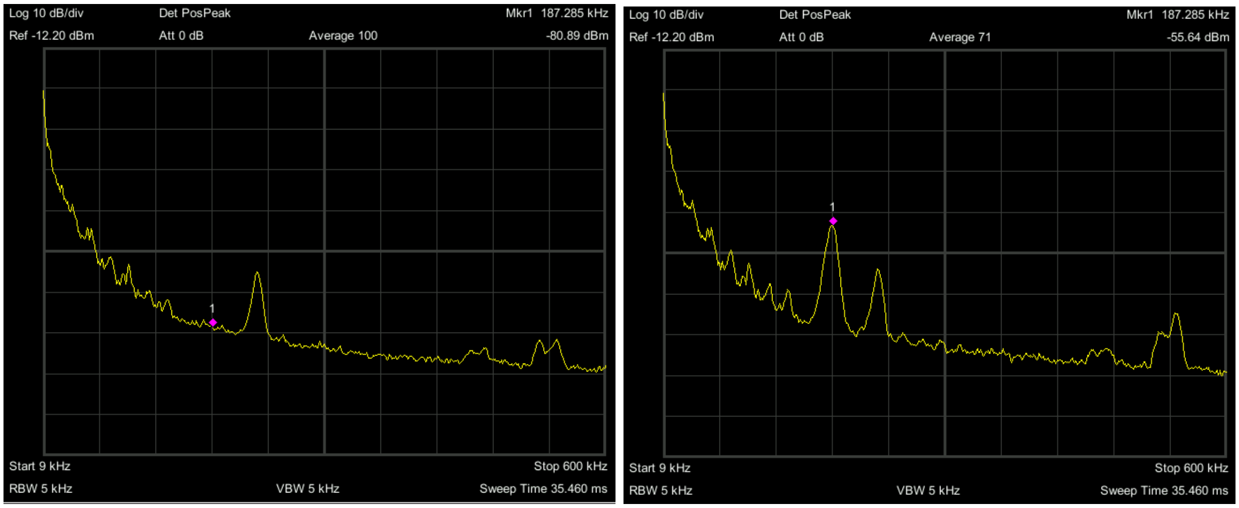

Figure 5: Images from the spectrum analyzer that show the difference between the spectrum of the laser [LEFT] and the spectrum of the laser and the interference pattern [RIGHT]. The beat frequency is marked with a 1.

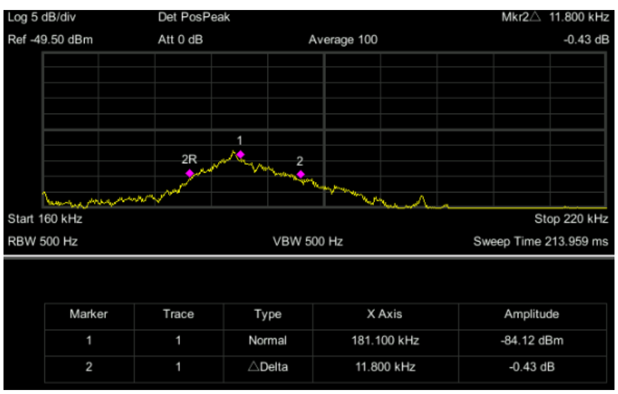

Figure 6: Data gathered from the spectrum analyzer collecting light from a He-Ne laser and light from the same laser scattered by a rotating wheel. Point 1 is the peak of the beat frequency, and points 2 and 2R are the half maximums. The distance between points 2 and 2R is the bandwidth of the beat frequency.

Fast Wheel

The direct measurement of the fast wheel was 256.0 ± 0.5 rad/s. The setup used had the following parameters and results:

$$\alpha = 0.059 \pm 0.005 \text{ rad}$$
$$\beta = 2.391 \pm 0.005 \text{ rad}$$
$$\nu_0 = (4.74 \pm 0.01) \times 10^{14} \text{ Hz}$$
$$\nu_{beat} = 1.5 \pm 0.1 \times 10^6 \text{ Hz}$$
$$r = 0.050 \pm 0.001 \text{ m}$$

Understanding the importance of these parameters will help in the results and in creating better experiments moving forward.

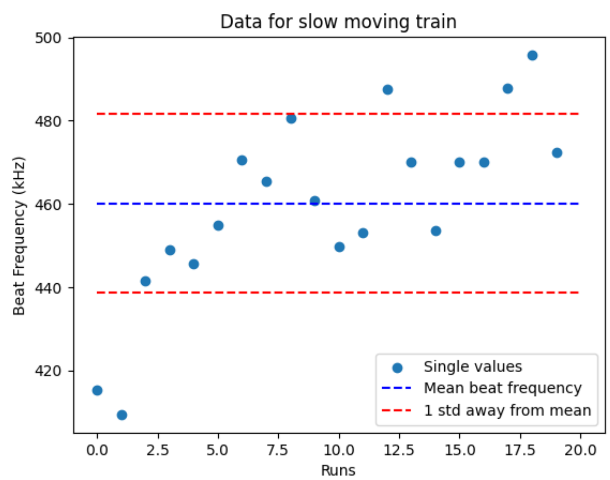

Figure 7: Beat frequency data gathered for the train experiment from the spectrum analyzer. All points are for different runs of the train moving at the same speed.

Results & Analysis

Slow Wheel

The light scattered off the wheel will be Doppler shifted twice, once as though the laser is the source and the wheel is the observer, then again as though the wheel is the source and the photodetector is the observer. Using this along with Eq. 1 and Eq. 2, the speed of the wheel will be

$$c\frac{\nu_{beat}}{2\nu_0 + \nu_{beat}} = v_{wheel} \tag{3}$$

where v_{wheel} is the longitudinal velocity of the wheel. The angular velocity of the wheel would come from $v_{wheel} = a\omega$ where ω is the angular frequency of the wheel.

The slow wheel is the most promising result out of all the values. For all five values of a, the uncertainty was all in the hundredth place. Figure 8 visually shows the error comparison. The final value was 1.24 ± 0.01 rad/s. This is the same as the speed found by taking the average speed over multiple runs.

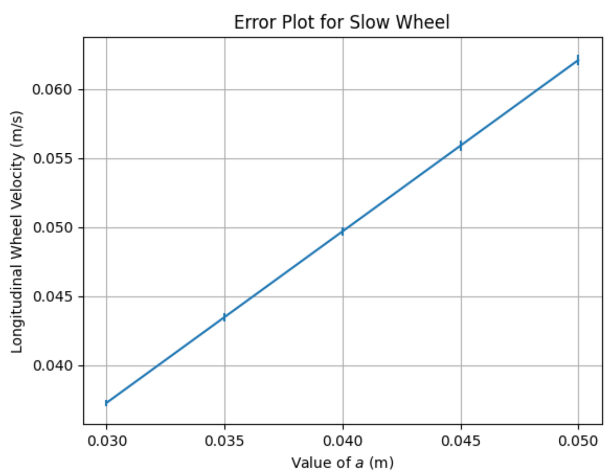

Figure 8: This is the error plot for the slow-moving wheel data. The error bars show three times the uncertainty to see them. The y-axis is the calculated velocity from Eq. 3. The x-axis is the a value. The slope of this plot is the angular frequency of the wheel ω.

Slow Train

Using the mean value of the ν_{beat} for the train (see Figure 7) and error propagation, the speed of the train is 0.146 ± 0.006 m/s. This is a 21% error when compared with the directly measured value. Our predicted standard deviation is also very tight, so if we take the value measured by taking multiple runs and averaging as the actual value, the Doppler effect measured speed is eight standard deviations from the actual value. This is probably due to the nature of the train that was used. The train was only able to run on about half a meter of track on the table where the Doppler experiment was taking place, so it might not have had enough time to reach its top speed. That might also explain why the spread seen in Figure 7 is so wide since any early or late measurement would return different results. The direct measurement was done where the train had more track, so it likely got up to a higher speed.

Fast Wheel

The equation for the angular speed of the wheel given the setup in Figure 4 is

$$\omega = \frac{c\nu_{beat}}{4\nu_0 r \sin(\beta)\sin\left(\frac{\alpha}{2}\right)} \tag{4}$$

where α is the angle between the beams hitting the wheel, β is the angle from the tangential surface of the wheel to the bisector of α, and r is the wheel's radius.

The resulting angular velocity of the wheel after error propagation is 220 ± 20 rad/s. This is two sigmas from the actual value of 256.0 ± 0.5 rad/s. This does support the idea that this method has promise but needs to be done with more care. One way to minimize error is to choose the setup to minimize the impact of uncertainty. When propagating error, the error in the α and β angles become meaningful. The part of the error that is related to β is:

$$\sigma_\omega = \sqrt{\frac{\sigma_\beta^2 c^2 \nu_{beat}^2 \cos^2(\beta)}{16\nu_0^2 r^2 \sin^4(\beta)\sin^2\left(\frac{\alpha}{2}\right)} + \ldots} \tag{5}$$

This value can be made smaller by choosing a setup that makes β as close to $\pi/2$ as possible. We cannot do the same with α because that would remove the purpose of the experiment.

Discussion

Although the results were not extremely accurate, this report shows that measuring the beat frequency between light that was Doppler shifted off a moving object can be used to find the speed of the object. The technique is best for stationary objects that are moving slowly but can be used elsewhere. In ideal circumstances, we showed that this method is extremely accurate and does not introduce a lot of additional uncertainty.

When the object is moving, the technique is still viable, but this report was inconclusive on whether or not it could have the same low uncertainty that was found for the stationary object. Future work should be done to see if this technique can measure the acceleration of a uniformly accelerating train. This will likely be more effective since it will not depend on when the beat frequency was found and is a more general question. While the results of the train were far in terms of standard deviation, there was only a 21% error between the actual measurements and the Doppler measurements.

The geometry of the experiment also matters. When using two beams of light and measuring the α and β angles, it is important to think about how one could position the angles in order to decrease uncertainty. Setting up the train with more track to ensure it reached the speed we needed it to be at would have also likely helped out with measurements. While versatile, if these conditions cannot be met, this is probably not the best technique for measuring speeds.

This report only tested the technique on longitudinal velocities, but the fast wheel method can also be used to find the transverse speed of an object. The resulting ν_{wheel} mentioned in the fast wheel section is the speed of the spot on the wheel that the beams are hitting, so this is the tangential velocity. This means that using the longitudinal Doppler effect, we can get the entire characterization of the instantaneous velocity of an object. While this report does not show strong evidence that it is universally accurate and precise, this technique is very valuable for tabletop experiments that need to find the velocity of objects. We recommend further tests to improve the understanding of how this can be used.

Conclusion

These results show that using the Doppler effect to measure the speed of tabletop objects is valid. The slow-moving wheel is a particularly good object to measure, but there is a promise to improve the fast-wheel measurement by setting it up with more favorable angles. This report has doubts that the measurement for the "actual" speed of the train is good enough to use and recommends others continue the work on the train measurements.

While this work shows that the Doppler effect can measure constant speeds, this method measures the instantaneous speed of objects. Using this method, researchers can get a function of the speed of an accelerating object with uncertainty proportional to the photodetector's and spectrum analyzer's sampling rates. Future work should be done to confirm or refute this claim.

References

[1] Mather J.C. Clampin M. et al. Gardner, J.P. The james webb space telescope. *Space Sci Rev 123, 485-606*, 2006.

[2] P. Hariharan. Chapter 6 - the laser as a light source. In P. Hariharan, editor, *Basics of Interferometry*, pages 45–55. Academic Press, San Diego, 1992.

[3] Antony Joseph. Chapter 2 - the history of measuring ocean currents. In Antony Joseph, editor, *Measuring Ocean Currents*, pages 51–92. Elsevier, Boston, 2014.

[4] E. Hecht. *Optics*. Pearson, 2012.